String theory

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Abstract. This is a rendering of a general talk on the state of String Theory given at the EPS-2003 conference. It is intended for a wide audience of experimental and theoretical physicists and emphasizes general ideas rather than technical aspects.

PACS. 11.25.-w Strings and branes

1 Introduction

String theory is the dominant framework for the construction of a unified quantum theory including gravity. In the last decade the theory underwent a major conceptual revolution whose consequences are still unfolding. In this article I review a selected number of recent research directions, presented against the background of various well established theoretical results.

The choice of topics is fairly subjective, and no attempt was made at collecting a complete reference list. General reviews of the subject with extensive lists of references can be found in [1,2,3].

2 What do we know?

Perhaps the most remarkable property of string theory is the emergence of gravity from a purely mechanical model. Starting from flat spacetime, quantizing a relativistic closed string without internal structure (i.e. a mathematically *thin* string), yields a quantum model of the graviton. This means that one finds universally a massless mode of spin 2 which couples at low energies according to General Relativity. At the same time, the interaction is soft at high energies, simply because highly energetic strings tend to grow in size and this smears the interactions.

String theory provides the only known quantum model of the graviton that is consistent at all energies, and hence it is the starting point of a theory of quantum gravity. In this sense, we may regard closed strings as the natural microscopic "excitations" of quantum spacetime. In perturbation theory, any string theory is characterized by a fundamental mass scale, M_s, that sets the intrinsic tension of the string. It is also characterized by a dimensionless coupling constant, $g_s < 1$, and together they determine

the Planck length (or Newton's constant) according to a relation of the form

$$G_{\mathrm{N}} \sim \ell_{\mathrm{P}}^2 \sim g_s^2 \, \ell_s^2 \,, \tag{1}$$

where $\ell_s = M_s^{-1}$ is the fundamental string length scale and ℓ_{P} is the Planck length. This formula is analogous to the expression for the Fermi constant of weak interactions in terms of the weak Yang–Mills coupling and the mass of the vector bosons:

$$G_{\mathrm{F}} \sim g_W^2 \, M_W^{-2} \,. \tag{2}$$

In fact, the logical status of both formulas is very similar, i.e. string theory is a *perturbative* smearing of the gravitational interaction.

String perturbation theory is a very tight formalism with a high degree of self-consistency. Heurisitcaly, the peculiar properties of perturbative strings are related to the topological character of the elementary string interactions; once the free propagation of a string is known (the free spectrum), the interactions are determined to all orders in the string coupling g_s.

Along with the graviton at the massless level, string models feature scalar, fermion, and gauge field excitations, the building blocks of the Standard Model (SM). Quasi-realistic models can be constructed at the qualitative level. This property has led to an interpretation of string theories as unification models including gravity. In fact, they incorporate many theoretical schemes that predate them, such as supersymmetry and extra dimensions of spacetime, and came to dominate the art of speculative model building for the last two decades.

2.1 Supersymmetry and stability

In many models, such as the simplest bosonic string, there are modes with $M^2 < 0$, i.e. tachyons that signify a dynamical instability of the spacetime. The only *generic* cure

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known for such instabilities is the assumption of spacetime supersymmetry. In supersymmetric models, the spectrum admits the action of a superalgebra

$$\{\,Q,Q\,\} \sim H + \ldots \tag{3}$$

where H denotes the Hamiltonian, and the vacuum is left invariant $Q\,|\text{vac}\rangle = 0$. Under such conditions, the operator M^2 is non-negative. This stability property is robust, in the sense that particular examples with approximate supersymmetry are stable. Approximate supersymmetry means that the scale of its breaking M_b is small compared to the string scale; $M_b \ll M_s$. In other words, low-energy supersymmetry appears as a "technical" requirement to ensure spacetime's local stability.

These considerations about the role of supersymmetry in the local stability of spacetime are based on experience with concrete models. Unfortunately, it has not been possible to prove a general theorem, much less to predict the numerical value of the ratio M_b/M_s. Hence, while the current understanding of string theory relies heavily on supersymmetry, we cannot quite say that strings predict low-energy supersymmetry, say at the TeV scale.

2.2 Vacuum structure and duality

A high degree of supersymmetry does play a pivotal role in the global structure of known string models. If the number of independent supercharges ranges between 8 and 32 the string models come in continuous families. The parameters of these vacua are called *moduli* and are interpreted in spacetime as massless scalar fields. In extreme regions of these *moduli spaces* of vacua, spacetime can be geometrically interpreted as the product of d-dimensional Minkowski spacetime times a compact manifold: $\mathbf{R}^d \times K$. Typically the dimension d_K of the compact smooth manifold is such that $d_K + d$ is either 10 or 11.

The low-energy effective theories on these manifolds of vacua are supergravity theories on $\mathbf{R}^d$ with scalar fields (the moduli) that are interpreted as the geometrical parameters of K (size and shape). The strength of the string coupling can also be understood as one such moduli, the dilaton ϕ, so that we can write an equation in all weakly-coupled string models that relates the expectation value of the dilaton with the coupling:

$$g_s = \exp\left(\langle\phi\rangle\right). \tag{4}$$

Therefore, we find continuous moduli spaces of vacua with extreme regions featuring a weakly-coupled description in terms of string perturbation theory (for $g_s \ll 1$,) and/or low-energy supergravity for $\ell_K \gg \ell_s$, ℓ_P, in terms of a characteristic length scale ℓ_K of the compact manifold K.

One of the great leaps forward in the 90's was the recognition that different string theories with *a priori* unrelated perturbative expansions could actually be mapped into one another by a discrete nonperturbative symmetry called *duality*, a generalization of known duality symmetries in electrodynamics and statistical mechanics. For example, the strong coupling limit of ten-dimensional

heterotic strings with gauge group $SO(32)$ turns out to be the weakly-coupled ten-dimensional type-I superstring theory. Since the type-I theory is a model of *open* unoriented strings and the heterotic model contains closed oriented strings, the two theories could not be more different at a perturbative level. Yet, they are related by a mapping of the form

$$g_s^{(H)} = 1/g_s^{(I)}\,.$$

Another famous example is the emergence of an eleven-dimensional vacuum with Poincaré symmetry as the strong coupling limit of ten-dimensional type-IIA strings.

Under these dualities, perturbative modes of one theory are transformed into nonperturbative states of another, such as solitons with mass proportional to $1/g_s$ or $1/g_s^2$. These solitons are visible in the supergravity Lagrangian as multidimensional generalizations of extremal black holes called p-branes. Particular states can be followed from weak to strong coupling when they are protected by supersymmetry, according to the so-called BPS phenomenon [4,5].

With many supercharges, it is possible that some finite energy states $|\psi_{\text{BPS}}\rangle$ are annihilated by a subset of the supercharges that leave the vacuum invariant. We refer to such charges as *unbroken*, Q_u, and they satisfy

$$Q_u\,|\psi_{\text{BPS}}\rangle = 0\,.$$

When these states admit a semiclassical description as solitons, the broken charges (the rest of them) generate Goldstone fermions that serve as fermionic collective coordinates for the low-energy dynamics of the soliton. At any rate, the dimensionality of the corresponding unitary representation is smaller than that of generic states, due to the vanishing of the Q_u on these states. This dimensionality being a discrete parameter, it cannot change by continuous deformations such as the variation of g_s or any other moduli. The result is that the BPS states can be followed around the supermoduli space and give information about its global structure, i.e. we can literally build a discrete "skeleton" of this moduli space. In this way very complicated moduli spaces with an action of large duality groups can be unravelled.

With 4 supercharges on the vacuum (the equivalent of $\mathcal{N} = 1$ in four dimensions) we can incorporate interesting features such as chiral fermion spectra at low energies. Most notably, exact $\mathcal{N} = 1$ vacua are expected to be generically *isolated*, with no moduli. Since moduli fields are always problematic for phenomenological models, this is a most interesting fact. Unfortunately, at the level of perturbation theory in g_s or ℓ_s/ℓ_K the $\mathcal{N} = 1$ vacua still come in moduli spaces, to be lifted only at the *nonperturbative* level. This task has been historically difficult for technical reasons and, to this date, no exact, isolated $\mathcal{N} = 1$ vacuum could be studied in any detail.

So far all the supersymmetric vacua considered contain a factor of Minkowski space. If we consider vacua with asymptocally *negative* curvature, we find geometries of the form $\text{AdS} \times K_\mathrm{E}$ with AdS denoting Anti-de Sitter

spacetime and K_{E} a compact Einstein manifold. These vacua are also isolated and admit a nonperturbative description in terms of the so-called AdS/CFT duality.

Finally, vacua with no supersymmetry remain largely inaccesible to precise theoretical analysis.

2.3 Impurities in spacetime: Singularities and branes

Each point in a supermoduli space of vacua represents a spacetime. In regions where supergravity is a good approximation, there is a geometrical description of the form $\mathbf{R}^d \times K$, with K a smooth complex manifold with appropriate holonomy group and metric properties (Calabi–Yau, K3, etc). However, as the moduli are varied, K may develop a variety of geometrical singularities whose physics may escape the effective supergravity description. Thus, the study of the structure of the supermoduli space is largely a question of singularity resolution. Since string theory is primarily a theory of gravity, the resolution of singularities is one of the basic problems that must solve.

It turns out that singularities on the appropriate (supersymmetric) manifolds K can be classified to a large extent and analyzed in quite physical terms. The various known mechanisms of singularity resolution in string theory are always associated to the emergence of extra light degrees of freedom localized at the singular locus. These extra light modes are often of topological nature. They might arise at a purely perturbative level, such as light winding modes at small circles or conical singularities in orbifolds, or they might be of nonperturbative origin, such as various wrapped solitonic branes. In many cases, the localized light modes are BPS-protected and we can write an exact low-energy effective theory that governs their dynamics.

The result is a physical resolution of the singularity when taking into account the dynamics of the light modes. Sometimes the singularity is just smoothed out by stringy fuzziness, such as the conical singularities of orbifolds. In other situations, a similar looking conical singularity (the conifold) develops a new branch of spacetime with nontrivial topological transitions between different manifolds. The large variety and richness of dynamical resolution of singularities has turned this problem into more of an art than a craft, the main limitation being the restriction to supersymmetric types of singularities.

The most interesting "impurities" of spacetime are the D-branes. They are submanifolds of spacetime defined by the condition that open strings can end on them. They are the stringy resolution of solutions of General Relativity that correspond to higher dimensional generalizations of extremal black holes. These impurities give a rationale for the existence of open string theories. One can say that while closed strings are the quantum excitations of the smooth part of spacetime, open strings are the quantum excitations of these particular impurities, the D-branes.

D-branes are at the core of most of the recent developments in string theory. Their most important property is the development of a rank-N nonabelian gauge symmetry when N D-branes sit on top of each other.

2.4 AdS/CFT and nonperturbative strings

The collective dynamics of a single Dp-brane is a theory of open strings with endpoints confined to the $p+1$ dimensional worldvolume. At long wavelengths this open string theory always contains a $U(1)$ gauge multiplet. If we accumulate N D-branes at a point in transverse space, an enhanced $U(N)$ symmetry develops. At the same time, the gravitational radius of the supergravity solution scales as

$$R \sim (g_s N)^{1/4}\, \ell_s\ . \tag{5}$$

Hence, in the limit $N \gg 1$, $g_s \ll 1$ with $g_s N \gg 1$ the gravitational radius stays much larger than the string scale and at low energies General Relativity provides a good description. From the point of view of the $U(N)$ gauge theory, $g_s N = g^2 N$ is the 't Hooft coupling of the $1/N$ expansion, so that the previous limit corresponds to the large-N expansion of the $SU(N)$ Yang–Mills theory with fixed and large 't Hooft coupling [6].

The celebrated AdS/CFT conjecture [7,8,3] states that the large-N dynamics of the gauge theory on the world-volume of the branes is equivalent to the gravitational description based on the near-horizon limit of the supergravity solution.

In the cases where the correspondence is well understood, the gauge theory has an ultraviolet fixed point of the renormalization group, which defines a conformal field theory (CFT). The corresponding dual geometry is asymptotic to Anti-de Sitter space (AdS) times a compact Einstein manifold: AdS $\times K_{\mathrm{E}}$. In the simplest example, we have a duality between type-IIB strings on AdS$_5 \times \mathbf{S}^5$ with N units of Ramond–Ramond flux on the sphere, and the large-N dynamics of $\mathcal{N} = 4$ super Yang–Mills theory with gauge group $SU(N)$.

The spacetime where the CFT is defined can be characterized as the conformal boundary of the AdS gravitational background. For example, the conformal boundary of AdS$_5$ is the conformal class of four-dimensional Minkowski space. More explicitly, the correspondence states that the generating functional of CFT correlation functions equals the quantum partition function of the string theory with given boundary conditions:

$$\left\langle \exp\left(\int_{\partial M} J\,\mathcal{O} \right) \right\rangle = \exp\left(-I_{\mathrm{eff}}\left[\phi \to \phi_{\partial M} = J \right] \right)\ . \tag{6}$$

In this expression the gravitational effective action is evaluated as a function of the boundary values of fields ϕ at the boundary ∂M of the bulk spacetime $M = \mathrm{AdS} \times K_{\mathrm{E}}$.

Since the CFT is a standard quantum field theory without gravity, we may be able to define it nonperturbatively. In this way, a strong version of the AdS/CFT correspondence provides a nonperturbative definition of string theory in certain spaces that are asymptotic to AdS space.

2.5 Holography and the entropy test

The AdS/CFT correspondence offers the most explicit realization of the holographic principle [9,10]. According

to this principle, quantum states associated to a region of space can be written in terms of degrees of freedom on the boundary of this region. This idea is based on the physics of black holes, the most compact form of matter, and in particular the peculiar scaling of the Bekenstein–Hawking entropy with the area of the event horizon. According to these ideas, the bulk of spacetime is a purely semiclassical concept, a sort of WKB artifact with a limited range of validity.

The high energy spectrum of finite-energy excitations in a gravity theory is given by black holes. The largest black holes supported by AdS_{d+1} space have an entropy of order

$$S_{\mathrm{BH}} = \frac{A_{\mathrm{Horizon}}}{4\,G_{\mathrm{N}}} = C\ (M\,R)^{\frac{d-1}{d}}\ , \qquad (7)$$

where M is the mass of the black hole and R the radius of curvature of the AdS space. This is exactly the scaling of the thermal entropy of CFT defined on a spatial sphere $\mathbf{S}^{d-1}$ of radius R, provided we identify M with the CFT energy. Thus, the density of states at high energy supports the idea of holography: there are enough states in a CFT defined on the boundary of AdS to account for all finite-energy excitations of gravity. In the cases where the corresponding CFT entropy could be exactly computed, it was found in complete agreement with the Bekenstein–Hawking formula, down to the factor of 1/4 [11].

This is arguably the most important quantitative test ever made in string theory, and in some sense it is the first time that the theory meets an unambiguous numerical check. The importance of this success can hardly be understated. Its main limitation: it holds only for black holes that can be regarded as excitations of supersymmetric vacua. Thus, even if the black holes themselves are not BPS states, they sit in a Hilbert space that does admit the action of an exact supersymmetry algebra.

2.6 Fundamental strings as QCD strings

The quantum equivalence between a four-dimensional $SU(N)$ gauge theory and a string theory with coupling proportional to $1/N$ is an old hypothesis and one of the conceptual venues towards the understanding of quark confinement [6]. What is remarkable in this case is the emergence of a ten-dimensional "fundamental" string theory, with gravity and all. A priori, the QCD string could be anticipated to be some sort of "effective" or "fat" string with a thickness of order $\Lambda_{\mathrm{QCD}}^{-1}$ in terms of the non-perturbative dynamical scale Λ_{QCD}. Instead, we find a "thin" string with gravity in extra dimensions with peculiar negative curvature. The background is such that integrating out the extra dimensions generates the appropriate degree of nonlocality to induce the "thickness" of the QCD string in the physical four dimensional spacetime.

Therefore, the AdS/CFT correspondence offers the first nontrivial example of large-N string in the sense of [6]. In some cases it was possible to build models with

many properties of QCD, such as confinement, gluon condensates in the vacuum, etc. However, all these models are defined by a soft breaking of supersymmetric and conformal models at some scale M_b. Let $\lambda = g^2 N$ be the value of the 't Hooft coupling at the scale M_b. QCD is obtained by taking $\lambda \ll 1$ so that a large hierarchy of order

$$\frac{\Lambda_{\mathrm{QCD}}}{M_b} \sim \exp\left(-C/\lambda\right) \qquad (8)$$

is generated. Unfortunately, in all known examples the limit $\lambda \ll 1$ is techically difficult in terms of the string theory. Standard approximate methods only apply to $\lambda \gg 1$ and in this strong coupling regime one has

$$\frac{\Lambda_{\mathrm{QCD}}}{M_b} \sim \lambda^{\alpha} \gg 1\ , \qquad (9)$$

violating the standard scaling of an asymptotically free theory. Thus, a constructive procedure exists to approach QCD on the string side, starting with some well defined models, but unfortunately the result stays out of calculational reach.

2.7 $\mathcal{N} = 1$ vacua and phenomenology

String model building has long known good approximations of the (supersymmetric) Standard Model, including features such as the gauge group, generations and chiral representations. The pool of available options was considerably increased by the consideration of geometrical models of the form $\mathbf{R}^4 \times K^*$, where the asterisk stands for the possibility of decorating the compact manifold with various "impurities" in the form of branes and fluxes.

This enhanced diversity comes at the price of removing many of the "model independent" predictions of old models based on the weakly coupled heterotic string on Calabi–Yau manifolds. One of the most quantitative such predictions was the relation between the Grand Unification scale M_{GUT} and Newton's constant,

$$G_{\mathrm{N}} > \frac{\alpha_{\mathrm{GUT}}^{4/3}}{M_{\mathrm{GUT}}^2}\ , \qquad (10)$$

where $\alpha_{\mathrm{GUT}} \sim 1/25$ is the value of the gauge couplings at the unification scale. The bound on G_{N} comes out too large by a factor of about 400, that must be blamed on threshold corrections. In the last few years it was recognized that localizing the SM gauge interactions on singularities of K^*, notably branes of various kinds, one could remove this constraint, at the price of losing one prediction [12,13]. The general tree-level formula for the string mass scale

$$M_s \sim \left(\frac{g_s^2}{G_{\mathrm{N}}\,\mathrm{Vol}\,(K^*)}\right)^{1/8} \qquad (11)$$

allows us to lower M_s down to a few TeV, at the price of increasing the size of the compact manifold. These large extra dimensions are transverse to the branes that confine the SM fields, so that they are largely invisible to SM

processes. This is the much studied scenario of large extra dimensions [14]. One can in principle build models with a string scale anywhere in the range (see for example [15])

$$\text{few} \quad \text{TeV} \ < M_s < 10^{18} \ \text{GeV} \ . \tag{12}$$

In fact, the relevant geometrical parameter of K^* is not necessarily its volume, it can also be the radius of (negative) curvature, leading to the *warped* scenario of [16, 17].

The emphasis on chirality reduces the choices to models with $\mathcal{N} = 1$ supersymmetry. In perturbation theory in a given string theory, such models have exactly massless moduli and exact supersymmetry. Thus, the perturbative approximation yields a moduli space of vacua with $\mathcal{N} = 1$ supersymmetry in four dimensions, in spite of the expectation that *exact* vacua should be isolated.

The moduli fields appear in four dimensions as gravitationally coupled scalars, and they are very constrained experimentally. In fact, they are the major embarrasment for string models that approximate qualitative features of the Standard Model. To date, no realistic vacua without moduli could be constructed explicitly. Most of the work has proceeded by trying to lift the moduli and break supersymmetry at the same time, all in the perturbative shores of the moduli space where we can justify the calculations. Thus, the so-called "moduli problem" has been tied to the problem of supersymmetry breaking, in part for technical reasons. The lack of a satisfactory solution of these problems stands as the main obstacle in rendering string phenomenology predictive.

3 A selection of recent developments

With no claim of completeness, we will mention some of the most significant trends of theoretical research in string theory. On the one hand, they are triggered by the problems posed by the AdS/CFT correspondence, both in its application to quantum gravity and to the problem of the QCD string. One the other hand, we have witnessed a revival of the study of time-dependent backgrounds with applications to cosmology as well as important progress in the classic problem of moduli stabilization.

3.1 Towards the QCD string

An very active area of research is the ongoing effort to bring the AdS/CFT models closer to real QCD by gradually lifting the constraints of conformal symmetry and supersymmetry.

As explained above, AdS/CFT models with soft breaking at scale M_b can be studied in the supergravity approximation in an expasion in powers of M_b/Λ_{QCD}. In order to invert this expansion parameter and approach the physical regime of pure non-supersymmetric Yang–Mills theory one must solve the string theory exactly in the $N \rightarrow \infty$ limit.

Although this feat remains beyond our present capabilities, interesting progress has been achieved recently in certain kinematical limits.

One possibility is to study the original AdS/CFT model in the limit of large R-charge [18]. Out of the global $SO(6)$ R-symmetry of the $\mathcal{N} = 4$ theory we may select a $U(1)$ subgroup and consider the limit of large charge. In the gravitational description, this introduces an infinite boost of the $\text{AdS}_5 \times \mathbf{S}^5$ geometry along an equator of $\mathbf{S}^5$. In this limit the gravitational description simplifies and we can solve exactly the tree-level string theory in the light-cone gauge. Thus we can extend the holographic correspondence beyond the BPS limit, provided we zoom into this sector of the total Hilbert space. The result is a rich generalization of AdS/CFT with interesting questions about the rules of holography and the role of string field theory.

Another interesting kinematical limit is that of large spin in the physical space. One considers the gauge theory on a spatial 3-sphere and takes the limit of large angular momentum J on an equator of $\mathbf{S}^3$. On the AdS side, it is then possible to identify special solitonic string states dual to operators satisfying [19]

$$\Delta - J \sim \log \ J \ , \tag{13}$$

where Δ stands for the anomalous dimension of the operator. The left-hand side of this equation is nothing but the *twist* of the operator, in the language of deep inelastic scattering. In fact, the logarithmic behaviour is a famous consequence of asymptotic freedom in perturbation theory [20].

It is rather intriguing to see this logarithm arising here from a purely geometrical calculation. It shows that focusing on special operators of large quantum numbers one can hope to bridge the gap between the weak and strong 't Hooft coupling.

3.2 The question of background independence

One crucial lesson of current nonperturbative definitons of string theory is their dependence on asymptotic boundary conditions. For asymptotically AdS spaces we can define appropriate boundary conditions that specify a Hamiltonian. For asymptotically flat spaces are just able to define an S-matrix that might be calculable in principle through a limit of AdS/CFT or perhaps the matrix theory of [21]. What could be the analogous structures relevant to spacetimes with closed spatial sections and/or cosmological singularities remains a mystery. For many years it was assumed that string field theory would hold the answer by providing a nonperturbative, background independent formulation of string theory. However, the developments centered about the realizations of holography (matrix theory and the AdS/CFT correspondence) severely question this hope. In fact, all evidence based on existing models tend to discourage the idea of background independence.

The simplest example of a background not falling in the understood categories is de Sitter space, the maximally

symmetric space of constant positive curvature. It breaks supersymmetry and cannot be recovered as a smooth compactification of higher-dimensional supergravity [22]. Allowing "impurities" in the compact manifold, such as D-branes and fluxes, it seems possible to construct *metastable* vacua with positive cosmological constant, i.e. metastable de Sitter bubbles [23].

Thus, one possibility is that de Sitter space can only be defined as a metastable resonance in the S-matrix of an asymptotically flat, supersymmetric, vacuum [24], but there are at least two other more radical proposals.

One is the so-called dS/CFT correspondence of [25], a sort of analytic continuation of AdS/CFT with a different physical interpretation in which cosmological time is identified with a renormalization group flow between conformal fixed points. Yet another one uses a radical interpretation of holography to claim that quantum de Sitter space has a finite-dimensional Hilbert space with most states localized at the observer's event horizon [26]. In this proposal the cosmological constant is an input related to the dimension of the Hilbert space rather than a calculable parameter.

Such a diversity of proposals that are well motivated, and yet so different at the conceptual level, show how fascinating this problem is, but they also reveal the primitive stage of our understanding.

3.3 Time dependent backgrounds and cosmological singularities

Time dependent backgrounds in string theory have been comparatively less studied beyond the supergravity approximation. In perturbation theory, the corresponding world-sheet conformal field theories are difficult to analyze. Despite these problems, interesting cosmological backgrounds exist which are based on non-compact coset models such as the classic model of [27]. Many physical aspects of these spacetimes have been studied recently [28], although the computation of the S-matrix beyond the one-particle scattering is still a notorious challenge.

Time-dependent orbifolds introduced in [29] are more amenable to analytic treatment and were extensively studied as toy models of pulsating universes (see for example the recent review [30]). The main result of these studies is negative, in the sense that back-reaction gets out of control of perturbation theory near the singularity. Very general arguments support the idea that these cosmological singularities are ultimately as hard as generic black hole singularities [31].

At a nonperturbative level, all the dilemmas afflicting the quantum mechanics of de Sitter space come back, in an even more agressive incarnation, since the singularities may deprive us from smooth asymptotic regions where the specification of the Hilbert space could be easier.

In general, the resolution of spacelike singularities in string theory is still uncharted territory. The great progress in the resolution of static (i.e. timelike) singularities is largely a consequence of supersymmetry and duality, while the big bang of a FRW model or the singularity of a black hole feature a maximal violation of supersymmetry.

Nevertheless, important lessons for cosmological singularities lie hidden in the AdS/CFT correspondence. Since large AdS black holes can be realized as thermal states of the CFT, it should be possible to extract information about the internal singularity from the thermal correlation functions of appropriate operators. The difficulty in doing so is our poor understanding of the holographic map beyond very symmetric or generic states. This is a fascinating (albeit difficult) set of problems whose exploration is only beginning [32, 33, 34, 35].

Recently, a large effort is being devoted to the understanding of the simpler problem of time-dependent openstring backgrounds. These can be interpreted as dynamical processes involving unstable branes (D-brane decay) or systems of branes (D-brane antiD-brane annihilation). These systems have even been proposed as semi-realistic cosmological models in the context of the large extra dimensions scenario [36]. See [37] for a recent summary of applications to inflationary models.

Perturbatively in g_s, these processes are determined by boundary perturbations of the world-sheet conformal field theory [38]. Conversely, on the worldvolume of the branes we have the dynamics of a tachyonic mode rolling down a potential. This is a characteristic problem of open string field theory and with this motivation it has been much studied. Recently, semiclassical computations of the energy loss to closed strings have been performed [39], and the issue of back reaction is tackled in the context of two-dimensional toy models [40].

3.4 Flux compactifications and vacua statistics

As pointed out above, the existence of massless moduli fields coupled gravitationally stands out as a clearly unphysical feature of supersymmetric string models of low-energy phenomenology. For semi-realistic $\mathcal{N} = 1$ models, such a defect is presumably an artifact of perturbation theory. Yet, the problem of *moduli stabilization* stands as a classic difficulty in rendering any model quantitative.

Upon supersymmetry breaking, the problem becomes more complicated, and ties up with the thorny issue of the cosmological constant. Typical effective potentials for moduli, based for example on scenarios of gaugino condensation, show runaway behaviour unless one fine-tunes the dynamics to achieve a stable vacuum at weak coupling. In this case, one naturally finds masses for the moduli and cosmological constant controlled by the supersymmetry breaking scale $M_b \sim 1$ TeV. Even postponing the problem of the vacuum energy, moduli masses around the TeV scale cause notorious problems to the standard theory of nucleosynthesis [41].

Recently, significant progress was achieved in the purely technical problem of stabilizing moduli. In fashionable models of the form $\mathbf{R}^4 \times K^*$, where the compact space K is decorated with branes, orbifold singularities and trapped magnetic fluxes, there are a huge number of discrete choices for K^*, and it was found that the effective potential on $\mathbf{R}^4$ depends on these quantum numbers in an intrincate

way (see the contribution of T. Taylor to this conference for more details).

For example, consider N units of magnetic flux

$$\oint_{\Sigma} F = N \, , \qquad (14)$$

trapped on a submanifold Σ inside K^*. In normal models there are dozens of independent fluxes of this type. The contribution to the effective potential scales like

$$V_{\text{flux}} \sim \int_{K^*} |F|^2 \sim \frac{\text{Vol}\,(K)\,N^2}{\text{Vol}\,(\Sigma)^2} \, . \qquad (15)$$

On the other hand, N' wrapped branes on a cycle Σ' contribute

$$V_{\text{brane}} \sim N'\,T_{\text{brane}} \, \cdot \, \text{Vol}\,(\Sigma') \, . \qquad (16)$$

Again, a typical scenario may have hundreds of such independent wrapping modes. Combining many fluxes and branes and including gravitational corrections one is able to derive an effective potential that fixes most moduli for each choice of the set of discrete quantum numbers N_i.

In this method, one literally stabilizes the internal manifold K^* in a "mechanical" fashion, equilibrating tension force from the wrapping with magnetic repulsion from the trapped fluxes. To be precise, the modulus corresponding to the overall size, $\text{Vol}\,(K^*)$, remains unfixed in these models, and one must invoke other mechanisms such as the old gaugino condensation to complete the job [23].

One interesting aspect of these methods is their generality, potentially applying to many model-building scenarios. For example, one can think of fixing the moduli with masses $m_\phi \gg$ TeV, thus alleviating the cosmological moduli problems.

However, perhaps the most striking aspect of this scenario is its new angle on the cosmological constant problem. The contribution of fluxes to the vacuum energy depends on the discrete numbers N_i as

$$V_{\text{min}} = \Lambda_b + \sum_{i=1}^{n_f} C_i\,N_i^2 \, , \qquad (17)$$

where Λ_b stands for the contribution from other sources and n_f is the number of relevant independent fluxes (easily of $O(100)$). Assuming that Λ_b is negative, the vacua with cosmological constant in the physical range, $\Lambda_{\text{ph}} \pm \delta\Lambda$, are the solutions of the discrete equation

$$|\Lambda_b| + \Lambda_{\text{ph}} - \delta\Lambda < \sum_{i=1}^{n_f} C_i\,N_i^2 < |\Lambda_b| + \Lambda_{\text{ph}} + \delta\Lambda \, . \qquad (18)$$

The number of vacua in the spherical band of width $2\delta\Lambda$ grows exponentially with n_f; we have a quasicontinuous spectrum of vacua that is known as the *discretuum* [42].

In this scheme, we are guaranteed of finding an astronomical number of vacua with cosmological constant within acceptable limits. Recent estimates yield exponentially large numbers [43]. In principle, a small fraction of these

will have other desirable features, such as large mass hierarchies and correct particle content, and one can hope that the SM will occur "in the list". However, with such large numbers of vacua involved, one must wonder whether the scheme is at all testable, even in principle.

4 Concluding remarks

Our survey shows that enormous progress was achieved in elucidating the conceptual status of string theory as a model of quantum gravity. There is a global picture of models with extended super-Poincaré symmetry and a fairly explicit nonperturbative formulation of the theory on asymptotically AdS spaces. This formulation conforms to the general ideas of holography and the succesful calculation of the Bekenstein–Hawking entropy for certain black holes stands as the main quantitative test of these ideas.

The current frontier of development lies in the extension of these ideas to non-supersymmetric spacetimes, notably backgrounds with cosmological interpretation, a notoriously hard challenge. This is arguably the area of string theory in most urgent need for improvement, because even the simplest of examples, de Sitter space, poses a formidable theoretical challenge.

Of course, de Sitter space is also quickly becoming a phenomenological urgency, given the apparent measurement of a strictly positive cosmological constant [44] and the mounting evidence in favour of an early inflationary era in our universe [45]. On the positive side, this means that string theory and quantum gravity could be closer than expected to experimental tests.

The AdS/CFT correspondence also provides the first examples of large-N gauge strings in four dimensions, with nontrivial dynamical properties such as confinement. The succesful lifting of the constraints of supersymmetry and conformal symmetry remains the main obstacle in the approach to real QCD, a difficult but extremely important problem.

The reformulation of the unification paradigm in terms of strings provides a global framework for virtually all past scenarios of physics beyond the SM. In recent years, thanks to the versatility of D-branes and the understanding of duality symmetries, the number of quasi-realistic models has increased considerably, at the price of losing some old "model-independent" predictions. It is now possible to entertain many models with fundamental scale as low as a few TeV, changing the traditional perspective on the mass hierarchy problems. The stabilization moduli in a physically acceptable way remains as a major problem in which we are seeing considerable progress. The picture of a *discretuum* of vacua gradually emerges, to the disconfort of many, that would like a more predictive scenario.

It should be mentioned that phenomenological model building is implicitly based on the construction of effective potentials over perturbative moduli spaces. In the light of the issues of background dependence discussed above, it is possible or even likely that the dynamical choice of a physical, cosmological vacuum is a problem beyond our present theoretical tools [46].

One physical property pervades the whole theoretical building of string theory as we know it: supersymmetry. A radical but well motivated view would hold that supersymmetry is not just an offspin of string theory, but rather lies at its very foundation. Although supersymmetry at the TeV scale is not a solid prediction, finding experimental evidence in its favour is of the utmost importance for string theory.

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